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On the "uniform" observability of discrete-time nonlinear systems

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***Abstract*— In constructing an observer for a discrete-time nonlinear system, the system is commonly required to satisfy a certain kind of "uniform" observability condition, that is, the state should always be reconstructible from observation windows of a specific length, irrespective of the values of the state and inputs. In this paper, it is proved that this "uniform" requirement is unnecessary in the sense that if the initial state and inputs are on a compact set, then the "uniform" observability is derived from its non-uniform counterpart.**

I. INTRODUCTION

The observability and observers of nonlinear systems have been studied extensively for several decades([4], [6]–[9], [13]–[15], [18]–[20], [26], [27], [31], [32], [34], to name a few). Research efforts have been directed mainly toward continuous-time systems, in which a geometric approach has established significant results [15], [20], [26], but considerable efforts have been made to obtain similar results for discrete-time systems [10], [21], [30]. However, a "geometric-like" approach has been less successful for discrete-time nonlinear systems; hence, in recent years, an alternative approach based on numerical optimization, which is closely related to model predictive control [22], [23], has been attracting attention [3], [24], [25], [29].

A numerical optimization approach for designing discrete-time nonlinear observers requires a certain kind of "uniform observability." Roughly speaking, a discrete-time nonlinear system is uniformly observable provided that, if the observer selects a sufficiently wide observation window of the output sequence, then for any admissible state and input, the state is uniquely reconstructible from the observed output sequence. However, the requirement that a common window width is usable "uniformly" to any state and input appears restrictive and requires some justification.

The objective of this paper is to show that the requirement on the "uniformity" is not restrictive, in the sense that, if the initial state and inputs are inside a compact set, the state is distinguishable non-uniformly (the observation window required to distinguish two states may have different widths for different pairs of states), and for each state and input, there is an observation window of some width that satisfies the observability rank condition, then the system is uniformly observable.

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II. NOTATIONS AND DEFINITIONS

Consider a discrete-time nonlinear system of the form

$$\begin{aligned} x(t+1) &= f(x(t), u(t)), \\ y(t) &= h(x(t)), \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state with $x(0) \in \Omega_X$, $u(t) \in \Omega_U \subset \mathbb{R}^{n_u}$ is the control input, and $y(t) \in \mathbb{R}^{n_y}$ is the output. We assume that Ω_X and Ω_U are compact, f is C^1 on $\mathbb{R}^n \times \mathcal{N}(\Omega_U)$, where $\mathcal{N}(\Omega_U)$ is an open set containing Ω_U, and h is C^1 on $\mathbb{R}^n$. By $\phi_l(t, x(t); u)$, we denote the solution of (1) at the time instant $l+t$ initialized with $x(t)$ at the time instant t, that is, $\phi_0(t, x(t); u) = x(t)$, and for $l > 0$, $\phi_l(t, x(t); u) = f(\phi_{l-1}(t, x(t); u), u(t+l-1))$. Similarly, we define a l-length observation map (window) by

$$\eta_l(t, x(t); u) = \begin{pmatrix} h(x(t)) \\ \dots \\ h(\phi_{l-1}(t, x(t); u)) \end{pmatrix}. \tag{2}$$

In considering the observability, only the possibility of reconstructing $x(t)$ from a finite observation window $\eta_l(t, x(t); u)$ is important, and the initial time t is immaterial. Therefore, for simplicity of notation, we let $t = 0$ and abbreviate $\phi_l(0, x(0); u)$ and $\eta_l(0, x(0); u)$ as $\phi_l(x(0); u)$ and $\eta_l(x(0); u)$, respectively *.

We denote the finite sequence of inputs $(u(t), \dots, u(l))$ by $u[t, l]$ and the infinite sequence $(u(t), u(t+1), \dots)$ by $u[t, \infty]$. The countable product of Ω_U is denoted by $\prod_{\mathbb{N}} \Omega_U$.

A sequence $(x_1, x_2, \dots)$ is denoted by $(x_k)_{k\in\mathbb{N}}$. Note that, in this notation, the suffix has no relation with the time.

The symbol $\mathcal{N}(\cdot)$ denotes an open neighborhood.

In the sequel, it is sometimes necessary to consider the autonomous system corresponding to (1), in which $u(t)$ is identically zero. In autonomous cases, we omit the second arguments of $\phi(\cdot)$ and $\eta(\cdot)$ and denote them by $\phi_l(x(0))$ and $\eta_l(x(0))$, respectively.

Several concepts related to some "uniform" observability of discrete-time nonlinear systems have been used in the literature; however the following two notions are the most fundamental (different authors refer to them by different names, for both autonomous and non-autonomous systems, under slightly different conditions and contexts).

Definition 1 [2], [4], [5], [7], [11], [14], [16], [17], [23], [25], [30] *The system (1) is said to be uniformly observable on* Ω_X

*In this notation, the set Ω_X is replaced with $\phi_t(0, \Omega_X; u)$, the image of the set Ω_X. This set is an image of a compact set by a continuous map and is also compact.

with respect to all admissible inputs if $\exists N > 0$, $\forall u[0,\infty] \in \prod_{\mathbb{N}} \Omega_U$, *the map* $\eta_N(x; u)$ *is injective as a function of* x.

Definition 2 [4], [5], [7], [10], [14], [21], [25] *The system (1) is said to satisfy the uniform observability rank condition on* Ω_X *with respect to all admissible inputs if* $\exists N > 0$, $\forall x \in \Omega_X$, $\forall u[0,\infty] \in \prod_{\mathbb{N}} \Omega_U$, $\mathrm{rank}\ (\partial\eta_N/\partial x)|_{(x;u)} = n$.

Some authors define a certain kind of uniform observability by using a class $\mathcal{K}$ function (see e.g., [1], [3], [24], [29]). We do not employ it because our definitions are more directly related to classical definitions of observability [33].

The non-uniform counterparts of Definition 1 and Definition 2 are as follows.

Definition 3 *The system (1) is said to be distinguishable on* Ω_X *with respect to all admissible inputs if* $\forall x_1, x_2 \in \Omega_X$, $\forall u[0,\infty] \in \prod_{\mathbb{N}} \Omega_U$, $x_1 \neq x_2 \Rightarrow \exists N_{x_1,x_2,u}, \eta_{N_{x_1,x_2,u}}(x_1; u) \neq \eta_{N_{x_1,x_2,u}}(x_2; u)$.

Definition 4 *The system (1) is said to satisfy the observability rank condition on* Ω_X *with respect to all admissible inputs if* $\forall x \in \Omega_X$, $\forall u[0,\infty] \in \prod_{\mathbb{N}} \Omega_U$, $\exists N_{x,u}$, $\mathrm{rank}\ (\partial\eta_{N_{x,u}}/\partial x)|_{(x;u)} = n$.

In the following section, we prove that distinguishability combined with the observability rank condition is equivalent to uniform observability combined with the uniform observability rank condition.

III. MAIN RESULTS

We first consider the autonomous case, which is necessary in proving the non-autonomous counterpart.

Proposition 5 *The autonomous system corresponding to (1) satisfies the following two conditions:*

ZUO1) $\exists N_A > 0$, $\eta_{N_A}(x)$ *is injective on* Ω_X *as a function of* x,

ZUO2) $\exists N_B > 0$, $\forall x \in \Omega_X$, $\mathrm{rank}\ (\partial\eta_{N_B}/\partial x)|_x = n$,

if and only if the following two conditions are fulfilled:

ZO1) $\forall x_1, x_2 \in \Omega_X$, $x_1 \neq x_2 \Rightarrow \exists N_{x_1,x_2} > 0, \eta_{N_{x_1,x_2}}(x_1) \neq \eta_{N_{x_1,x_2}}(x_2)$,

ZO2) $\forall x \in \Omega_X$, $\exists N_x > 0$, $\mathrm{rank}\ (\partial\eta_{N_x}/\partial x)|_x = n$.

Proof. Because the "only if" part is obvious, we prove the "if" part only.

First, we prove that ZO2) implies ZUO2). If $\mathrm{rank}\ (\partial\eta_{N_x}/\partial x)|_x = n$ for some $N_x > 0$, then $\exists B_x \in \mathcal{N}(x)$, $\forall x' \in B_x$, $\mathrm{rank}\ (\partial\eta_{N_x}/\partial x)|_{x'} = n$. For each $x \in \Omega_X$, select a pair (N_x, B_x) that satisfies the above requirement. Because $\{B_x : x \in \Omega_X\}$ covers Ω_X and Ω_X is compact, there is a finite subcollection $\{B_{x_1}, \ldots, B_{x_m}\}$ of $\{B_x : x \in \Omega_X\}$ that also covers Ω_X. From the definition of the observation window (2), if $N_1 > N_2$, then η_{N_1} is of the following form:

$$\eta_{N_1}(x) = \begin{pmatrix} \eta_{N_2}(x) \\ h(\phi_{N_2}(x)) \\ \cdots \\ h(\phi_{N_1-1}(x)) \end{pmatrix}.$$

Therefore, by letting $N_B = \max\{N_1, \ldots, N_m\}$, we obtain ZUO2).

Next, we prove that ZO1) and ZO2) imply ZUO1) by contradiction. Suppose that ZUO1) is not true, that is, $\forall k \in \mathbb{N}$, $\exists (x_{1,k}, x_{2,k}) \in \Omega_X \times \Omega_X$, $x_{1,k} \neq x_{2,k}$ and $\eta_k(x_{1,k}) = \eta_k(x_{2,k})$. Let $((x_{1,k}, x_{2,k}))_{k\in\mathbb{N}}$ be the sequence of pairs that satisfies the above requirement. Because Ω_X is compact, $\Omega_X \times \Omega_X$ is also compact, therefore, $((x_{1,k}, x_{2,k}))_{k\in\mathbb{N}}$ has an accumulation point (x_{1*}, x_{2*}). Select a subsequence of $((x_{1,k}, x_{2,k}))_{k\in\mathbb{N}}$ that converges to (x_{1*}, x_{2*}). By a slight abuse of notation, we denote the subsequence itself by the same symbol $((x_{1,k}, x_{2,k}))_{k\in\mathbb{N}}$.

We prove that $x_{1*} = x_{2*}$ by contradiction. Suppose that $x_{1*} \neq x_{2*}$. From ZO1), $\exists N_*$, $\eta_{N_*}(x_{1*}) \neq \eta_{N_*}(x_{2*})$. Because $\eta_{N_*}(x)$ is C^1 and hence continuous, there exist some $B_1 \in \mathcal{N}(x_{1*})$ and $B_2 \in \mathcal{N}(x_{2*})$ with $B_1 \cap B_2 = \emptyset$ that satisfy $\forall x'_1 \in B_1$, $\forall x'_2 \in B_2$, $\eta_{N_*}(x'_1) \neq \eta_{N_*}(x'_2)$. On the other hand, $\exists \bar{k}$, $\forall k \geq \bar{k}$, $x_{1,k} \in B_1$ and $x_{2,k} \in B_2$. Hence, $\forall k \geq \max\{\bar{k}, N_*\}$, $\eta_k(x_{1,k}) = \eta_k(x_{2,k})$, whereas $\eta_{N_*}(x_{1,k}) \neq \eta_{N_*}(x_{2,k})$, hence $\eta_k(x_{1,k}) \neq \eta_k(x_{2,k})$, a contradiction. Therefore, $x_{1*} = x_{2*}$.

Let $x_{1*} = x_{2*} = x_*$. Recall that we have already obtained ZUO2), that is, $\mathrm{rank}\ (\partial\eta_{N_B}/\partial x)|_{x_*} = n$. Consider the "tail" subsequence of $((x_{1,k}, x_{2,k}))_{k\in\mathbb{N}}$ with $k \geq N_B$. Because $\eta_{N_B}(x)$ is differentiable,

$$\begin{aligned} 0 &= \eta_{N_B}(x_{2,k}) - \eta_{N_B}(x_{1,k}) \\ &= (\partial\eta_{N_B}/\partial x)|_{x_{1,k}} (x_{2,k} - x_{1,k}) + \nu(x_{1,k}, x_{2,k}) \end{aligned} \tag{3}$$

with $\nu(x_{1,k}, x_{2,k}) = o(\|x_{2,k} - x_{1,k}\|)$, that is, $\forall \varepsilon > 0$, $\exists \delta_1(\varepsilon) > 0$, $\|x_{2,k} - x_{1,k}\| < \delta_1(\varepsilon)$ implies that $\|\nu(x_{1,k}, x_{2,k})\| < \varepsilon\|x_{2,k} - x_{1,k}\|$. Since $\partial\eta_{N_B}/\partial x$ is continuous, $\forall \varepsilon > 0$, $\exists \delta_2(\varepsilon) > 0$, $\|x_* - x_{1,k}\| < \delta_2(\varepsilon)$ implies that

$$\| (\partial\eta_{N_B}/\partial x)|_{x_*} - (\partial\eta_{N_B}/\partial x)|_{x_{1,k}} \| < \varepsilon.$$

Moreover, because $\mathrm{rank}\ (\partial\eta_{N_B}/\partial x)|_{x_*} = n$, for some $\mu > 0$,

$$\| (\partial\eta_{N_B}/\partial x)|_{x_*} (x_{2,k} - x_{1,k})\| \geq \mu\|x_{2,k} - x_{1,k}\|.$$

Because $((x_{1,k}, x_{2,k}))_{k\in\mathbb{N}}$ converges to (x_*, x_*), by selecting a sufficiently large k, we may assume that $\|x_{2,k} - x_{1,k}\| < \delta_1(\mu/4)$ and $\|x_* - x_{1,k}\| < \delta_2(\mu/4)$. Then,

$$\begin{aligned} &\| (\partial\eta_{N_B}/\partial x)|_{x_{1,k}} (x_{2,k} - x_{1,k}) + \nu(x_{1,k}, x_{2,k})\| \\ &\geq \| (\partial\eta_{N_B}/\partial x)|_{x_{1,k}} (x_{2,k} - x_{1,k})\| - (\mu/4)\|x_{2,k} - x_{1,k}\| \\ &\geq \| (\partial\eta_{N_B}/\partial x)|_{x_*} (x_{2,k} - x_{1,k})\| - (\mu/2)\|x_{2,k} - x_{1,k}\| \\ &\geq (\mu/2)\|x_{2,k} - x_{1,k}\|, \end{aligned}$$

contradicting (3). □

Remark 6 One may wonder if ZUO2) implies ZUO1) (semi)globally, which is true at least locally. This type of assertion is called the "real Jacobian conjecture," and to the best of the author's knowledge, there is no definite answer. Several interesting special cases in which this assertion is true are known [12], whereas there is a 2-dimensional example in which this assertion is false [28].

Next, we consider the case of non-autonomous system.

Theorem 7 *The system (1) satisfies the conditions of Definition 1 and Definition 2 if and only if it satisfies the conditions of Definition 3 and Definition 4.*

Proof. Because the "only if" part is obvious, we prove the "if" part only.

We first prove that the condition of Definition 2 is satisfied if the condition of Definition 4 is satisfied. Let us introduce the relative topology on $\prod_{\mathbb{N}} \Omega_U$ as the subspace of $\prod_{\mathbb{N}} \mathbb{R}^{n_u}$ with the product topology. By Tychonoff's theorem, $\prod_{\mathbb{N}} \Omega_U$ is compact.

From assumption, for any $x \in \Omega_X$ and $u[0,\infty] \in \prod_{\mathbb{N}} \Omega_U$ (we abbreviate the latter as u if there is no risk of confusion), there is an $N_{x,u} \in \mathbb{N}$ that satisfies $\operatorname{rank}\, (\partial \eta_{N_{x,u}}/\partial x)|_{(x;u)} = n$. Note that $\eta_{N_{x,u}}(x;u)$ is a function of x and $u[0, N_{x,u}-1]$ and does not depend on $u[N_{x,u},\infty]$. Let $B_{u,N_{x,u}}$ be an open neighborhood of $u[0, N_{x,u}-1]$ as a subset of the finite product space $\prod_{N_{x,u}} \Omega_U$, and $B^{\infty}_{u,N_{x,u}} = B_{u,N_{x,u}} \times \prod_{\mathbb{N}} \Omega_U$. Then, $B^{\infty}_{u,N_{x,u}}$ is an open neighborhood of u. Because $\partial \eta_{N_{x,u}}/\partial x$ is a continuous function of x and $u[0, N_{x,u}-1]$, $\exists B_x \in \mathcal{N}(x)$, $\exists B^{\infty}_{u,N_{x,u}}$, $\forall x' \in B_x$, $\forall u' \in B^{\infty}_{u,N_{x,u}}$, $\operatorname{rank}\, (\partial \eta_{N_{x,u}}/\partial x)|_{(x';u')} = n$. For each x and u, select the triplet $(N_{x,u}, B_x, B^{\infty}_{u,N_{x,u}})$ that satisfies the above requirement, and let $B_{x,u} = B_x \times B^{\infty}_{u,N_{x,u}}$. Consider the collection of all pairs $\{(N_{x,u}, B_{x,u}) : x \in \Omega_X, u \in \prod_{\mathbb{N}} \Omega_U\}$. Because $\Omega_X \times \prod_{\mathbb{N}} \Omega_U$ is compact, $B_{x,u}$ is open and $\{B_{x,u} : x \in \Omega_X, u \in \prod_{\mathbb{N}} \Omega_U\}$ covers $\Omega_X \times \prod_{\mathbb{N}} \Omega_U$, there is a finite subcollection $\{B^{(1)}_{x,u}, \ldots, B^{(m)}_{x,u}\}$ that also covers $\Omega_X \times \prod_{\mathbb{N}} \Omega_U$. Thus, the condition of Definition 2 is fulfilled by letting $N = \max\{N^{(1)}_{x,u}, \ldots, N^{(m)}_{x,u}\}$.

Next, we prove that the condition of Definition 1 is satisfied. Note that Proposition 5 is still true for a fixed u. Therefore, $\forall u$, $\exists N_u > 0$, $\eta_{N_u}(x;u)$ is injective on Ω_X. By letting $N_u = \max\{N, N_u\}$ if necessary, we may assume that $(\partial \eta_{N_u}/\partial x)|_{(x;u)}$ is of full rank.

We prove that $\exists B_{u,N_u} \in \mathcal{N}(u[0, N_u-1])$, $\forall u' \in B^{\infty}_{u,N_u}$, $\eta_{N_u}(x;u')$ is injective on Ω_X by contradiction. Suppose that this assertion is not true, that is, $\forall B_{u,N_u} \in \mathcal{N}(u[0, N_u-1])$, $\exists u' \in B^{\infty}_{u,N_u}$, $\exists x_1, x_2 \in \Omega_X$ with $x_1 \neq x_2$, $\eta_{N_u}(x_1;u') = \eta_{N_u}(x_2;u')$. Let $B^{\infty}_{u,N_u,k} = \{u' \in \prod_{\mathbb{N}} \Omega_U : \|u'(j) - u(j)\| < 1/k, j = 0, \ldots, N_u - 1\}$. Select $(x_{1,k}, x_{2,k}) \in \Omega_X \times \Omega_X$ and $u'_k \in B^{\infty}_{u,N_u,k}$ that satisfy the above requirement. Let an accumulation point of $((x_{1,k}, x_{2,k}))_{k\in\mathbb{N}}$ be (x_{1*}, x_{2*}). Then, there is a subsequence of $((x_{1,k}, x_{2,k}, u'_k[0, N_u-1]))_{k\in\mathbb{N}}$ that converges to $(x_{1*}, x_{2*}, u[0, N_u-1])$ (note that the finite sequence $u'_k[0, N_u-1]$ converges to $u[0, N_u-1]$ by construction). By construction, $\eta_{N_u}(x_{1*};u) = \eta_{N_u}(x_{2*};u)$, and because $\eta_{N_u}(x;u)$ is injective, $x_{1*} = x_{2*}$. Let $x_* = x_{1*} = x_{2*}$. By the same argument as the proof of the latter half of Proposition 5, this contradicts that assumptions that $\eta_{N_u}(x;u)$ is C^1 and $(\partial \eta_{N_u}/\partial x)|_{(x_*;u)}$ is of full rank.

Let $\{(N_u, B^{\infty}_{u,N_u}) : u \in \prod_{\mathbb{N}} \Omega_U\}$ be a collection of pairs such that $\forall u' \in B^{\infty}_{u,N_u}$, $\eta_{N_u}(x;u')$ is injective on Ω_X. Because $\{B^{\infty}_{u,N_u} : u \in \prod_{\mathbb{N}} \Omega_U\}$ is an open cover of $\prod_{\mathbb{N}} \Omega_U$ and $\prod_{\mathbb{N}} \Omega_U$ is compact, there is a finite subcollection $\{B^{\infty(1)}_{u,N_u}, \ldots, B^{\infty(m)}_{u,N_u}\}$ that covers $\prod_{\mathbb{N}} \Omega_U$. Then, by letting $N = \max\{N^{(1)}_u, \ldots, N^{(m)}_u\}$, we obtain the desired result. □

Remark 8 For discrete-time nonlinear systems which are equivalent to a specific canonical form, sharper results than Theorem 7 are known to hold [4], [5].

Example 1 Consider the following system:

$$\begin{aligned} x_1(t+1) &= f_1(x_1(t), x_2(t), u(t)), \\ x_2(t+1) &= f_2(x_1(t), x_2(t), u(t)), \\ y(t) &= x_1(t), \end{aligned} \tag{4}$$

where $u(t) \in \{0,1\}$ and

$$\begin{aligned} &f_1(x_1(t), x_2(t), u(t)) = \begin{cases} x_1(t)x_2(t) + 1 & \text{if } u(t) = 1, \\ x_2(t) & \text{if } u(t) = 0, \end{cases} \\ &f_2(x_1(t), x_2(t), u(t)) = x_2(t). \end{aligned} \tag{5}$$

Because the input space is discrete, the C^1 assumption on $f = (f_1, f_2)^T$ is reinterpreted as the continuous differentiability with respect to $(x_1(t), x_2(t))$.

Let the initial condition be (α_1, α_2). We see that (5) is distinguishable in the sense of Definition 3. Note that, by (5), $\forall t \geq 0$, $x_2(t) = \alpha_2$. If $u(0) = 0$, then $(y(0), y(1)) = (\alpha_1, \alpha_2)$, hence the state is distinguishable. Similarly, if $u(0) = 1$ and $u(1) = 0$, then

$$(y(0), y(1), y(2)) = (\alpha_1, \alpha_1\alpha_2 + 1, \alpha_2),$$

hence the state is distinguishable. Finally, if $u(0) = 1$ and $u(1) = 1$, then

$$(y(0), y(1), y(2)) = (\alpha_1, \alpha_1\alpha_2 + 1, \alpha_1\alpha_2^2 + \alpha_2 + 1).$$

This observation window is injective. For, if

$$\begin{aligned} &\left(\alpha_1, \alpha_1\alpha_2 + 1, \alpha_1\alpha_2^2 + \alpha_2 + 1\right) \\ &\quad = \left(\alpha'_1, \alpha'_1\alpha'_2 + 1, \alpha'_1(\alpha'_2)^2 + \alpha'_2 + 1\right), \end{aligned} \tag{6}$$

then $\alpha_1 = \alpha'_1$. If $\alpha_1 \neq 0$, the second equality of (6) implies that $\alpha_2 = \alpha'_2$. If $\alpha_1 = 0$, the third equality of (6) implies that $\alpha_2 = \alpha'_2$.

It is immediate that the observability rank condition of Definition 4 is also fulfilled by the same observation windows.

Therefore, by Theorem 7, (4) satisfies the conditions of Definition 1 and Definition 2, and is uniformly observable.

Remark 9 Systems given in the nonlinear observer canonical form

$$\begin{aligned} x_1(t+1) &= x_2(t) + g_1(x_1(t), u(t)), \\ &\cdots \\ x_{n-1}(t+1) &= x_n(t) + g_{n-1}(x_1(t), \ldots, x_{n-1}(t), u(t)), \\ x_n(t+1) &= g_n(x_1(t), \ldots, x_n(t), u(t)), \\ y(t) &= x_1(t) \end{aligned}$$

are automatically uniformly observable.

IV. Conclusion

In this paper, we have proved that, if the initial state and inputs are on a compact set, then the uniform observability is derived from the distinguishability of each pair of states together with the pointwise observability rank condition. Thus, the uniform observability condition that is frequently required in designing optimization-based nonlinear observers (typically the moving horizon observers) is in fact a natural requirement.

Similar and much more extensive results for continuous-time nonlinear systems may be found in [7], [14].

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