

# 工共 212 工業数学 IV

## 第 12 回

### 関数の整級数展開

#### 演習 12-1 解答 (1)

$$\begin{aligned} \frac{1}{0!} \text{Log } z|_{z=1} &= \boxed{0}, \\ (\text{Log } z)' &= \boxed{z^{-1}}, \quad \frac{1}{1!} (\text{Log } z)'|_{z=1} = \boxed{1}, \\ (\text{Log } z)'' &= \boxed{-z^{-2}}, \quad \frac{1}{2!} (\text{Log } z)''|_{z=1} = \boxed{-\frac{1}{2}}, \\ (\text{Log } z)''' &= \boxed{2z^{-3}}, \quad \frac{1}{3!} (\text{Log } z)'''|_{z=1} = \boxed{\frac{1}{3}}, \end{aligned}$$

#### 演習 12-1 解答 (2)

$$\begin{aligned} \text{Log } z &= \boxed{0} + \boxed{1}(z-1) + \boxed{\left(-\frac{1}{2}\right)}(z-1)^2 + \boxed{\frac{1}{3}}(z-1)^3 + \dots, \quad A = \boxed{1}, \\ \text{Log } z &= \boxed{(z-1) - \frac{1}{2}(z-1)^2 + \frac{1}{3}(z-1)^3 + \dots}, \\ &\text{一致する} \end{aligned}$$

#### 演習 12-1 解答 (3)

$$\begin{aligned} B &= \boxed{0}, \quad c_n = \frac{\boxed{(-1)^{n-1}}}{\boxed{n}}, \\ \lim_{n \rightarrow \infty} \left| \frac{\boxed{(-1)^{n-1}(n+1)}}{\boxed{(-1)^n n}} \right| &= \boxed{1} \end{aligned}$$

#### 演習 12-2 解答 (1)

$$\begin{aligned} \text{(A)} \quad \frac{1}{1-z} &= \sum_{n=0}^{\infty} z^n, \quad f(z) = -\sum_{n=0}^{\infty} z^{n-1} \\ &= \left(-\frac{1}{z} - 1 - z - \dots\right) \\ \text{(B)} \quad \frac{1}{1-\frac{1}{z}} &= \sum_{n=0}^{\infty} z^{-n}, \quad f(z) = \sum_{n=0}^{\infty} z^{-n-2} \end{aligned}$$

#### 演習 12-2 解答 (2)

$$\begin{aligned} \text{(C)} \quad \frac{1}{1+(z-1)} &= \sum_{n=0}^{\infty} (-1)^n (z-1)^n, \\ f(z) &= \sum_{n=0}^{\infty} (-1)^n (z-1)^{n-1} \end{aligned}$$

解答の書き方は以上と若干異なってもよい (たとえば  $z^{-1}$  を  $\frac{1}{z}$  と書くなど)