工共 212 工業数学 IV
第 8 回
演習

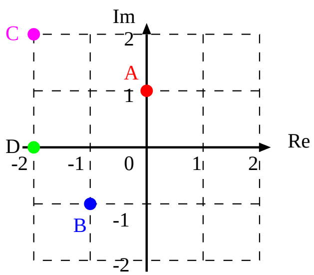
演習 8-1 解答

$$i\times i = (0,1)\times(0,1) = (\boxed{0}\,\boxed{0}-\boxed{1}\,\boxed{1}, \boxed{0}\,\boxed{1}+\boxed{0}\,\boxed{1})$$
$$= (\boxed{-1}, \boxed{0})$$

演習 8-2 解答

$$\alpha = 4+9i \quad \mathrm{Re}\,\alpha = \boxed{4}, \quad \mathrm{Im}\,\alpha = \boxed{9}$$
$$\alpha = 1.3 \quad \mathrm{Re}\,\alpha = \boxed{1.3}, \quad \mathrm{Im}\,\alpha = \boxed{0}$$
$$\alpha = \pi i \quad \mathrm{Re}\,\alpha = \boxed{0}, \quad \mathrm{Im}\,\alpha = \boxed{\pi}$$

演習 8-3 解答



演習 8-4 解答

1. $(-1+i)+(1-i) = \boxed{0}+\boxed{0}i \qquad (= -1-i)$
2. $(2-i)-(3-4i) = \boxed{-1}+\boxed{3}i$
3. $(1+i)(1+i) = \boxed{0}+\boxed{2}i$
4. $\dfrac{1}{1-i} = \dfrac{1+i}{(1-i)(1+i)} = \boxed{\dfrac{1}{2}}+\boxed{\dfrac{1}{2}}i$

演習 8-5 解答

- $-i$ の共役複素数は $\boxed{0}+\boxed{1}i$
- 1 の共役複素数は $\boxed{1}+\boxed{0}i$
- $\pi+\sqrt{2}i$ の共役複素数は $\boxed{\pi}+\boxed{-\sqrt{2}}i$

演習 8-6 解答

$$w_0 = \boxed{2}e^{i\boxed{(\pi/6)}} \qquad w_1 = \boxed{2}e^{i\boxed{(\pi/6+\pi/3)}}$$
$$w_2 = \boxed{2}e^{i\boxed{(\pi/6+2\pi/3)}} \qquad w_3 = \boxed{2}e^{i\boxed{(\pi/6+\pi)}}$$
$$w_4 = \boxed{2}e^{i\boxed{(\pi/6+4\pi/3)}} \qquad w_5 = \boxed{2}e^{i\boxed{(\pi/6+5\pi/3)}}$$

演習 8-7 解答

$$\lim_{z\to 0}\frac{4z-i}{iz+2} = \boxed{-i/2} \qquad \lim_{z\to 2i}\frac{3i-z}{3i} = \boxed{1/3}$$

演習 8-8 解答

$$(z^5)' = \boxed{5z^4}, \quad \left(\frac{z-\pi i}{z}\right)' = \frac{\boxed{\pi i}}{\boxed{z^2}},$$
$$(z^{-1})' = \boxed{-z^{-2}}$$

演習 8-9 解答

$z = x+iy$ とし, $f(z) = \overline{z}$ を $f(z) = u(x,y)+iv(x,y)$ の形に書き直すと $u(x,y) = \boxed{x}$, $v(x,y) = \boxed{-y}$ である. $\frac{\partial u}{\partial x} = \boxed{1}$, $\frac{\partial u}{\partial y} = \boxed{0}$, $\frac{\partial v}{\partial x} = \boxed{0}$, $\frac{\partial v}{\partial y} = \boxed{-1}$ であるから, $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ と$\boxed{\text{ならず}}$, $\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$ と$\boxed{\text{なる}}$. したがって, $f(z)$ は正則で$\boxed{\text{ない}}$.

演習 8-10 解答 (1)

$u(x,y) = \boxed{x^2 - y^2}$, $v(x,y) = \boxed{2xy}$, $\dfrac{\partial u}{\partial x} = \boxed{2x}$, $\dfrac{\partial u}{\partial y} = \boxed{-2y}$, $\dfrac{\partial v}{\partial x} = \boxed{2y}$, $\dfrac{\partial v}{\partial y} = \boxed{2x}$ となる.

演習 8-10 解答 (2)

$$f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = \boxed{2x} + i\boxed{2y}$$

$$f'(z) = \frac{1}{i}\left(\frac{\partial u}{\partial y} + i\frac{\partial v}{\partial y}\right) = \frac{1}{i}\left(\boxed{-2y} + i\boxed{2x}\right)$$

$$= \boxed{2x} + i\boxed{2y}$$

演習 8-11 解答 (1)

$c_0 = \boxed{1}$, $c_1 = \boxed{1}$, $\ldots$, $c_n = \boxed{1}$, $c_{n+1} = \boxed{1}$,

$\dfrac{c_n}{c_{n+1}} = \dfrac{\boxed{1}}{\boxed{1}} = \boxed{1}$, $\displaystyle\lim_{n\to\infty} \frac{c_n}{c_{n+1}} = \boxed{1}$, $r = \boxed{1}$.

$(1-z)f(z) = \boxed{1}$, $f(z) = \dfrac{\boxed{1}}{\boxed{1-z}}$.

演習 8-11 解答 (2)

$c_0 = \boxed{1/0!}$, $c_1 = \boxed{1/1!}$, $\ldots$,

$c_n = \boxed{1/n!}$, $c_{n+1} = \boxed{1/(n+1)!}$,

$\dfrac{c_n}{c_{n+1}} = \dfrac{\boxed{(n+1)!}}{\boxed{n!}} = \boxed{n+1}$,

$\displaystyle\lim_{n\to\infty} \frac{c_n}{c_{n+1}} = \boxed{\infty}$, $r = \boxed{\infty}$.