電 210 電気数学 IV
第 7 回
初等関数 (2)

演習 7-1 解答

$$
\begin{aligned}
e^{\ln 2 + i\frac{\pi}{3}} &= e^{\ln 2} e^{i\frac{\pi}{3}} \\
&= \boxed{2}\left(\cos\boxed{\frac{\pi}{3}} + i\sin\boxed{\frac{\pi}{3}}\right) \\
&= \boxed{1} + \boxed{\sqrt{3}}\,i
\end{aligned}
$$

演習 7-2 解答 (1)

$\sin 3i = \dfrac{e^{i(\boxed{3i})} - e^{-i(\boxed{3i})}}{2i} = \dfrac{e^{\boxed{-3}} - e^{\boxed{3}}}{2i}$,
$e^3 \simeq 20,\ e^{-3} \simeq 0$ と近似すると $\sin 3i \simeq \boxed{0} + \boxed{10}\,i$

演習 7-2 解答 (2)

$$
\begin{aligned}
\cos(\pi + 3i) &= \frac{e^{i(\boxed{\pi+3i})} + e^{-i(\boxed{\pi+3i})}}{2} \\
&= \frac{e^{\boxed{-3}+\boxed{\pi}i} + e^{\boxed{3}+\boxed{-\pi}i}}{2}
\end{aligned}
$$

演習 7-2 解答 (3)

オイラーの公式を使うと,

$$
\begin{aligned}
\cos(\pi + 3i) &= \tfrac{e^{\boxed{-3}}}{2}(\cos\boxed{\pi} + i\sin\boxed{\pi}) \\
&\quad + \tfrac{e^{\boxed{3}}}{2}(\cos\boxed{-\pi} + i\sin\boxed{-\pi})
\end{aligned}
$$

$e^3 \simeq 20,\ e^{-3} \simeq 0$ とすると

$$\cos(\pi + 3i) \simeq \boxed{-10} + \boxed{0}\,i$$

演習 7-3 解答 (1)

$\cosh 3 = \frac{e^{\boxed{3}}+e^{\boxed{-3}}}{2} \simeq \boxed{10}$, $\sinh 3 = \frac{e^{\boxed{3}}-e^{\boxed{-3}}}{2} \simeq \boxed{10}$ (より正確には $\cosh 3 = 10.067662$, $\sinh 3 = 10.017875$)

演習 7-3 解答 (2)

$$
\begin{aligned}
&\cosh^2 3 - \sinh^2 3 \\
&= \left(\frac{e^{\boxed{3}}+e^{\boxed{-3}}}{2}\right)^2 - \left(\frac{e^{\boxed{3}}-e^{\boxed{-3}}}{2}\right)^2 \\
&= \frac{1}{4}\left(\boxed{e^6} + \boxed{2} + \boxed{e^{-6}}\right) - \frac{1}{4}\left(\boxed{e^6} + \boxed{-2} + \boxed{e^{-6}}\right) \\
&= \boxed{1}
\end{aligned}
$$

演習 7-4 解答

$|i| = \boxed{1}$, $\mathrm{Arg}\, i = \boxed{\pi/2}$, $r = \boxed{1}$, $\theta = \boxed{\pi/2}$, $\log z = \ln r + i(\theta + 2n\pi)$ だから, $\log i = \boxed{0} + i\left(\boxed{\pi/2 + 2n\pi}\right)$ である.

演習 7-5 解答

$|e^4| = \boxed{e^4}$, $\mathrm{Arg}\, i = \boxed{0}$, $\arg i = \boxed{2n\pi}$ $(n \in \mathbb{Z})$, $e^4 = re^{i(\theta+2n\pi)}$ とすると $r = \boxed{e^4}$, $\theta = \boxed{0}$, $\log e^4 = \boxed{4} + i\left(\boxed{2n\pi}\right)$, $(n \in \mathbb{Z})$ であり, 一方 $\mathrm{Log}\, e^4 = \boxed{4} + i\left(\boxed{0}\right)$ である.